

Trajectory Tracking Control for Nonholonomic Wheeled Mobile Robots with External Disturbances and Parameter Uncertainties

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Abstract: The system studied in this research is a nonholonomic wheeled mobile robot (WMR) with three degrees of freedom (3-DOF). The goal is to design a fast end-sliding mode control for positioning and stabilizing the system's forward motion and angle. The methodology involves formulating the system's dynamic equations using Newton-Euler equations and then converting them into state-space form. The designed controller is then applied to the system, and external random noise disturbances are introduced via MATLAB simulations to evaluate the system's robustness against uncertainties and disturbances.

Keywords: Non-holonomic Wheeled Mobile Robot, Fast Terminal Sliding Mode Control, Sliding Surface, Lyapunov Function, Position Control.

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1. Introduction

Wheeled mobile robots (WMRs) have gained increasing popularity because of their amazing mobility and capabilities. Researchers and engineers around the world dedicate considerable focus on the autonomous supervision and command of these machines. WMRs are commonly utilized throughout a range of industries, including military applications space travel, and manufacturing. They are able to function in dangerous settings, and execute tasks independently without the involvement of humans.

On manufacturing lines, self-driving cars equipped with wheels are designed for rapid and precise movement in different directions. These robots offer cost-saving benefits while maintaining design speed and flexibility. They are typically Having two back wheels powered by independent motors and a free-moving front wheel, enabling omnidirectional movement. Their robust structure makes them ideal for various applications, as moving materials and products in factories.

Because there are many unknown parameters, it is difficult to adequately represent the three-wheeled mobile robot (3WMR) both kinematically and dynamically. It is classified as an underactuated system, it has less actuators than degrees of freedom, therefore. Furthermore, 3WMRs exhibit nonholonomic constraints, preventing them from moving sideways freely. Therefore, creating control algorithms is a difficult undertaking for 3WMRs, especially when putting controllers on actual devices because of mechanical uncertainties and environmental disruptions.

1.1 Related Work

It has been suggested the ABHSMC controller, a novel adaptive controller for three-wheel mobile robots (3WMR). This controller was created using a collaborative approach, a combination of a feedback controller and a hierarchical sliding mode con-

troller based on a radial basis function (RBF). It is noteworthy that, the RBF neural network shows a significant ability to calculate the model's uncertainty components and adjust its parameters in a methodical manner, which leads to the improvement of the output responses. A new navigation model called ABHSMC The ABHSMC navigation stack, timed elastic band (TEB) local scheduler, and controller are utilized to efficiently resolve the obstacle avoidance and tracking problem for a 3-wheel mobile robot. The Matlab/Simulink platform's implementation of the simulation results show that 3WMRs are able to adhere to the intended path exactly even when there are disruptions and modifications to the model. In addition, the reliability of the controller has been verified in the built Model of self-driving automobile. The experimental findings gained demonstrate that the suggested navigation structure may successfully guide the real car model to trace the intended course with a sufficiently small error and simultaneously avoid an unexpected barrier [1].

It has been presented an iterative learning control (ILC) strategy to solve the path tracking problem of wheeled mobile robots (WMR) based on dynamic model. In the previous study of trajectory tracking of WMRs, ILC was usually applied to the kinematic model of WMRs with the assumption that the desired velocity can be tracked immediately and directly. However, this assumption cannot be realized at all in the real world. Kinematic and dynamic models of WMR and a new combination of D-type ILC algorithm and dynamic model of WMR with stochastic boundary disturbances are presented. To analyze the convergence of the algorithm, a convention mapping method is adopted which shows that a designed controller can completely converge the velocity tracking errors to zero when the iteration times reach infinity. The simulation results show the effectiveness of the D-type ILC algorithm in the path tracking problem of WMRs, demonstrating the effectiveness and robustness of the algorithm under random boundary disturbance conditions. A comparative study between D-type ILC and neural network compound cosine function controller was conducted, which also shows the effectiveness of the ILC strategy [2].

It has been presented the speed and direction angle control of a wheeled mobile robot based on the sliding mode control technique based on the fractional order adaptive model. Considering the unique advantages of fractional arithmetic and adaptive model-based sliding mode PID control method, fractional order and adaptive model-based sliding mode PID control technique are combined and presented as a controller for the first time in the literature. effective for real-time control of wheeled mobile robot under wheeled external robot are proposed. In this proposed method, several critical issues are considered. First, a kinematic and dynamic model of the wheeled mobile robot is analyzed according to the uncertainties of the system. Second, fractional-order computations and model-based PID-type sliding mode control are formulated to realize chatter-free control, accurate path tracking response, finite-time convergence, and resistance for the wheeled mobile robot. Finally, an adaptive process is also employed to meet and overcome the unknown dynamics and uncertain parameters of the system, regardless of prior information about the uncertainties. The experimental results show that the proposed controller (fractional-order adaptive model-based sliding mode controller) has accurate track tracking performance, faster finite-time convergence, and also less speed error under external load when compared to the adaptive model-based PID sliding-mode control. It can be provided [3].

2. Methodology

Sliding mode controller is one of the most widely used controllers for studying and applying to nonlinear systems and processes. This acceptance is owing to both its naivety proportionality and its durability against system unsureness and exterior troubles. The unruly in its application is the phenomenon of states slipping on the slip surface or gossip. Sign functions are sometimes used in place of smooth functions like saturation functions or hyperbolic tangents in order to mitigate or eradicate this issue. In general, the

robustness of the sliding mode controller is higher than some controllers such as PID or LQR. Another advantage is that the control operation is performed on a nonlinear system and there is no need to linearize and approximate the system. Fast terminal sliding mode control can also be used. Fast terminal sliding mode control, which is the innovation of this research, can be used to ensure that the system states are placed on the slip surface in a short time. The advantages of this controller over the PD sliding mode control are that it not only increases the speed and accuracy of tracking the desired path, but also grow the robustness of the system of controls. Lastly, to assess the suggested control plan, a comparison between the error outcomes of the simulation of the above sliding mode methods with *MATLAB* software in the absence and presence of external disturbances is presented [4-6].

2.1 State Space Equations

First, we rewrite the dynamic equations governing the mobile robot [2]:

$$M(q) \ddot{q} + C(q, \dot{q})\dot{q} = E(q) T - A^T(q)\lambda \quad (1)$$

Equation (1) can be written as follows and used to control the system.

$$\ddot{q} = M^{-1}(q)(-C(q, \dot{q})\dot{q} + E(q) T - A^T(q)\lambda) \quad (2)$$

According to reference [1], the system dynamics equations are transformed as follows:

$$\begin{cases} \ddot{\theta} = b_1 u_1 \\ \ddot{x} = \frac{\lambda}{m} \sin(\theta) + b_2 u_2 \cos(\theta) \\ \ddot{y} = -\frac{\lambda}{m} \cos(\theta) + b_2 u_2 \sin(\theta) \end{cases} \quad (3)$$

We know that $\lambda = -m\ddot{\theta}(\dot{x} \cos(\theta) + \dot{y} \sin(\theta))$, $b_1 = \frac{L}{r_f}$, $b_2 = \frac{1}{rm}$, $u_1 = T_1 - T_2$, $T_1 = 0.5(u_1 + u_2)$, $T_2 = 0.5(u_2 - u_1)$ and $u_2 = T_1 + T_2$.

Now, considering the following relationships:

$$\begin{cases} X = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T = \\ \begin{bmatrix} x & \dot{x} & y & \dot{y} & \theta & \dot{\theta} \end{bmatrix}^T \\ U = [u_1 \ u_2]^T \end{cases} \quad (4)$$

which is the state space vector (in terms of the robot's position and velocity) and the torque vector (control input).

Therefore, we rewrite the set of equations (3) in the following form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x) + g_1(x)u_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x) + g_2(x)u_2 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = f_3(x) + g_3(x)u_1 \end{cases} \quad (5)$$

So that $f_1(x)$, $f_2(x)$, $f_3(x)$, $g_1(x)$, $g_2(x)$, $g_3(x)$ and have the following definitions.:

$$\begin{cases} f_1(x) = \frac{\lambda}{m} \sin(\theta), \ g_1(x) = b_2 \cos(\theta) \\ f_2(x) = -\frac{\lambda}{m} \cos(\theta), \ g_2(x) = b_2 \sin(\theta) \\ f_3(x) = 0, \ g_3(x) = b_1 \end{cases} \quad (6)$$

2.2 Control input design [7]

Here we will design the control input for two displacement states x & y & θ the front orientation angle of the mobile robot. The control diagram of the system is as follows in Figures 2.1 and 2.2:

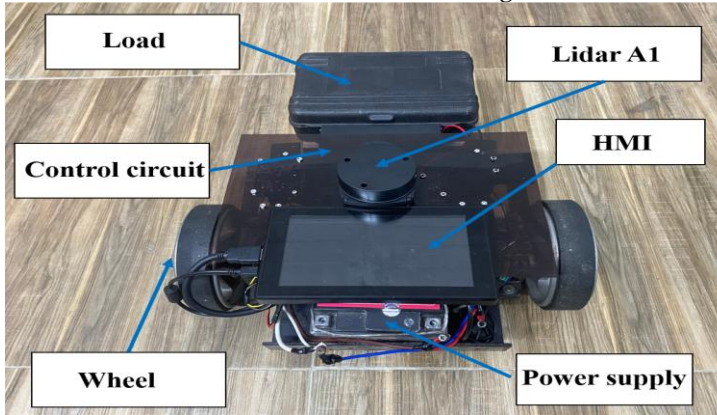


Figure 2.1. A model of a three-wheeled mobile robot

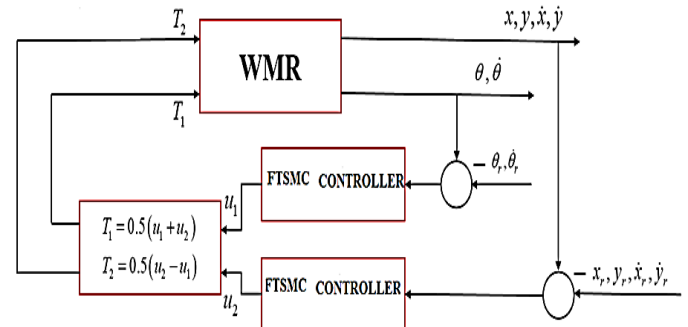


Figure 2.2. The control diagram of the system

Now, considering the values of the parameters, the initial state vector and the desired positions of the robot as displayed in the Table 1 below:

Table 1. Parameters considered for the simulation and command of the mobile, wheeled robot

Robot parameters	Values	Units
m	10	Kg
J	818/3	Kg/m ²
r	07/0	M
L	0/19	M

Favorable conditions

$$[0.2t + 1 \quad 1 + 0.25 \sin(0.2\pi) \quad [x_d \quad y_d \quad \theta_d]^T$$

Initial conditions of the robot

$$[x(0) \quad \dot{x}(0) \quad y(0) \quad \dot{y}(0) \quad \theta(0) \quad \dot{\theta}(0)]^T = [0 \quad 0 \quad 0.2 \quad 0 \quad 0.2\pi \quad 0]^T$$

First, we will design the controller for θ the angle. Considering the control method, which in this study is the fast terminal sliding mode control method, we first consider a nonlinear sliding surface called so that $S_1 \rightarrow 0$ the value of $x_5 \rightarrow x_{5d}$ and $\dot{x}_5 \rightarrow \dot{x}_{5d}$

therefore, the tracking of the angle and angular velocity is done. Therefore, we define the slip surface as equation (7):

$$S_1(x) = \dot{e}_{1x} + c_1 e_{1x} + c_2 (e_{1x})^{p/q} \quad (7)$$

So that c_1 and c_2 are positive parameters ($p < q$) and p and q are also integers. and $e_{1x} = x_5 - x_{5d}$ is, which is x_{5d} the desired state or reference of the robot orientation angle according to Table 1.

By deriving from (7) we arrive at the following relation:

$$\dot{S}_1(x) = \ddot{e}_{1x} + \dot{e}_{1x}(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1}) \quad (8)$$

Given that the slip the state variables are used to define the surface equation. and the desired state, the system will converge to the desired state on the slip surface. So, it can be said that the following relations hold on the slip surface:

$$\begin{cases} S_1 = 0 \\ \dot{S}_1 = 0 \end{cases} \quad (9)$$

Based on According to sliding mode theory, the control input has the following definition:

$$u_1 = u_{1eq} + u_{1s} \quad (10)$$

So that u_{1eq} the control component is equivalent and keeps the states on the slip surface u_{1s} and the switching component is responsible for directing the states towards the slip surface, which is actually responsible for stabilizing the system and is determined with the help of the Lyapunov stability criterion. To acquire the comparable control element, it is enough to put $\dot{S}_1 = 0$ and also take the help of equation (5):

$$\begin{aligned} \dot{S}_1(x) &= \ddot{e}_{1x} + \dot{e}_{1x}(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1}) = 0 \Rightarrow \\ (\dot{x}_6 - \dot{x}_{5d}) + (\dot{x}_5 - \dot{x}_{5d})(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1}) &= 0 \\ \Rightarrow u_{1eq} &= (\ddot{x}_{5d} + (x_6 - \dot{x}_{5d})(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1}))/g_3 \end{aligned} \quad (11)$$

Now evaluating the potential Lyapunov function as a positive definite function in the manner described below

$$V_1 = \frac{1}{2} S_1^2 > 0 \quad (12)$$

This function's derivative needs to be negative definite for asymptotic stability:

Using relation (11), and substituting into equation (13), we have:

$$\begin{aligned} \dot{V}_1 &= S_1 \dot{S}_1 = S_1(\ddot{e}_{1x} + \dot{e}_{1x}(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1})) = S_1((\dot{x}_6 - \dot{x}_{5d}) + (\dot{x}_5 - \dot{x}_{5d})(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1})) \\ &= S_1((g_3(u_{1eq} + u_{1s}) - \dot{x}_{5d} + (x_6 - \dot{x}_{5d})(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1})) < 0 \end{aligned} \quad (13)$$

$$\dot{V}_1 = S_1(g_3 u_{1s}) < 0 \quad (14)$$

Consequently, a good option for the switching control legislation to meet the requirement (16) is the following relation:

$$u_{1s} = (-k_1 \text{sign}(S_1) - k_2 S_1)/g_3 \quad (15)$$

Where k_1 and k_2 are positive real values. Substituting from equation (15) into equation (14), we have:

$$\dot{V}_1 = S_1 \left(\frac{g_3(-k_1 \text{sign}(S_1) - k_2 S_1)}{g_3} \right) = -k_1 S_1 \text{sign}(S_1) - k_2 S_1^2 < 0 \quad (16)$$

By combining equations (11) and (15) and substituting into relation (10), the general robust control equation for the fast terminal sliding mode to be applied to the wheeled mobile robot to track the desired reference path will be obtained:

$$u_1 = (\ddot{x}_{5d} + (x_6 - \dot{x}_{5d})(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1}))/g_3 + |(-k_1 \text{sign}(S_1) - k_2 S_1)/g_3| \quad (17)$$

Now we will design the controller for the displacements x and y . Since we want to control these two states with one input (u_2). Therefore, first we consider two nonlinear slip

surfaces named S_2 and S_3 such that with the value of $S_2 \rightarrow 0$ and $x_1 \rightarrow x_{1d}$ and $\dot{x}_1 \rightarrow \dot{x}_{1d}$ with the value of $S_3 \rightarrow 0$ $x_3 \rightarrow x_{3d}$ and $\dot{x}_3 \rightarrow \dot{x}_{3d}$. Therefore, linear position and velocity tracking is performed. For this purpose, we define the slip surfaces as equation (18):

$$\begin{cases} S_2(x) = \dot{e}_{2x} + c_3 e_{2x} + c_4 (e_{2x})^{p/q} \\ S_3(x) = \dot{e}_{3x} + c_5 e_{3x} + c_6 (e_{3x})^{p/q} \end{cases} \quad (18)$$

So that c_2 up to c_6 to the positive parameters and q and p ($p < q$) are also integers. And $e_{2x} = x_1 - x_{1d}$ and $e_{3x} = x_3 - x_{3d}$, which is x_{1d} the desired state or reference of the x robot movement and x_{3d} the desired state or reference y of the robot movement according to Table 1. Now, considering that we have only one control input in this part, we define a slip surface as a combination of S_2 and S_3 :

$$S_4(x) = \lambda_1 S_2(x) + \beta_1 S_3(x) \quad (19)$$

so that λ_1 and β_1 are positive parameters. To acquire the comparable control element, it is sufficient to set and also use relations (5) and (18):

$$\begin{aligned} \dot{S}_4(x) &= \lambda_1 \dot{S}_2(x) + \beta_1 \dot{S}_3(x) = \\ &\lambda_1 (\ddot{e}_{2x} + c_3 \dot{e}_{2x} + c_4 p/q \dot{e}_{2x} (e_{2x})^{(p/q-1)}) + \\ &\beta_1 (\ddot{e}_{3x} + c_5 \dot{e}_{3x} + c_6 p/q \dot{e}_{3x} (e_{3x})^{(p/q-1)}) = 0 \Rightarrow \\ &\lambda_1 (f_1 + g_1 u_{2eq} - \ddot{x}_{1d} + (x_2 - \dot{x}_{1d})(c_3 + c_4 \frac{p}{q} (e_{2x})^{\frac{p}{q}-1}) + \\ &\Rightarrow \beta_1 (f_2 + g_2 u_{2eq} - \ddot{x}_{3d} + (x_4 - \dot{x}_{3d})(c_5 + c_6 \frac{p}{q} (e_{3x})^{\frac{p}{q}-1}) = 0 \end{aligned} \quad (20)$$

Which u_{2eq} by solving the above equation we have:

$$u_{2eq} = \frac{-\lambda_1 (f_1 - \ddot{x}_{1d} + (x_2 - \dot{x}_{1d})(c_3 + c_4 \frac{p}{q} (e_{2x})^{\frac{p}{q}-1}) - \beta_1 (f_2 - \ddot{x}_{3d} + (x_4 - \dot{x}_{3d})(c_5 + c_6 \frac{p}{q} (e_{3x})^{\frac{p}{q}-1}))}{\lambda_1 g_1 + \beta_1 g_2} \quad (21)$$

Now evaluating the potential *Lyapunov* function as a positive definite function in the manner described below:

$$V_2 = \frac{1}{2} S_4^2 > 0 \quad (22)$$

Using relation (20) and the fact that the derivative for asymptotic stability must be negative definite, we have:

$$\begin{aligned} \dot{V}_2 &= S_4 \dot{S}_4 = S_4 (\lambda_1 (f_1 + g_1 (u_{2eq} + u_{2s}) - \ddot{x}_{1d} + (x_2 - \dot{x}_{1d})(c_3 + c_4 \frac{p}{q} (e_{2x})^{\frac{p}{q}-1}) + \beta_1 (f_2 \\ &+ g_2 (u_{2eq} + u_{2s}) - \ddot{x}_{3d} + (x_4 - \dot{x}_{3d})(c_5 + c_6 \frac{p}{q} (e_{3x})^{\frac{p}{q}-1})) < 0 \end{aligned} \quad (23)$$

Using relation (21), and substituting into equation (23), we have:

$$\dot{V}_2 = S_4 ((\lambda_1 g_1 + \beta_1 g_2) u_{2s}) < 0 \quad (24)$$

Consequently, a good option for the switching control legislation to meet the requirement (16) is the following relation

$$u_{2s} = (-k_3 \text{sign}(S_4) - k_4 S_4) / (\lambda_1 g_1 + \beta_1 g_2) \quad (25)$$

where k_3 and k_4 are positive real values. Substituting from equation (25) into equation (24), we have:

$$\dot{V}_2 = -k_3 S_4 \text{sign}(S_4) - k_4 S_4^2 < 0 \quad (26)$$

By combining equations (21) and (25) and substituting into relation (10), the general robust control equation for fast terminal sliding mode to be applied to a wheeled mobile robot to track the desired linear position and velocity reference path will be obtained:

$$u_2 = \frac{-\lambda_1(f_1 - \ddot{x}_{1d} + (x_2 - \dot{x}_{1d})(c_3 + c_4 \frac{p}{q} (e_{2x})^{\frac{p}{q}-1}) - \beta_1(f_2 - \ddot{x}_{3d} + (x_4 - \dot{x}_{3d})(c_5 + c_6 \frac{p}{q} (e_{3x})^{\frac{p}{q}-1})}{\lambda_1 g_1 + \beta_1 g_2 + (-k_3 \text{sign}(S_4) - k_4 S_4)/(\lambda_1 g_1 + \beta_1 g_2)} \quad (27)$$

3. Results and Discussion

3.1. Software simulation with MATLAB software

3.1.1. Fast terminal sliding mode control with sign function

Now, considering the simulation time $T_{sim} = 25$ s and sampling time constant $T_s = 0.001$ and the parameter values according to Table 1 are. Taking into account the control input that was created and their application in Matlab software, the graphs shown below are taken out [8, 9]:

In addition, to evaluate the performance of the control strategy used, in each graph its performance is compared with the classical sliding mode PD control, as shown in Figure 3.1. It is observed that the position and Θ angle tracking for PD sliding mode control and the proposed method are well performed, but the position tracking for PD sliding mode control has diverged from 15 seconds onwards, while the fast terminal sliding mode control has performed all tracking well, as shown in Figure 3.2

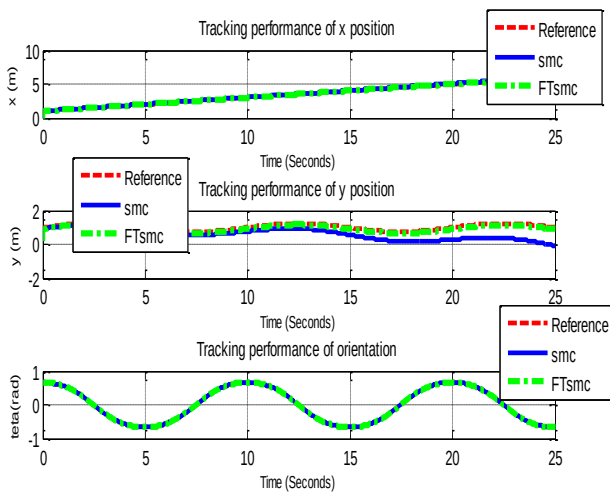


Figure 3.1. Tracking the paths related to the positions and angles of a mobile, wheeled robot with the sign function

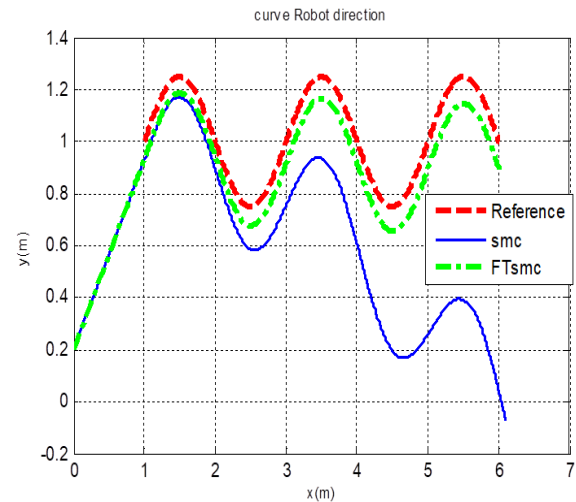


Figure 3.2. Robot tracking on the screen $x - y$ with the mark function

It is observed that the sinusoidal path tracking in the two-dimensional plane for PD sliding mode control deviates from the path after some time, while for fast terminal sliding mode control it is accompanied by some persistent error but always remains convergent. In Figure 3.3, it is observed that in the control effort diagrams, the unwanted oscillations at the first input for the PD sliding mode are very large. In Figure 3.4, it is observed that in the slip surface diagrams, the unwanted oscillations at the first slip surface for the PD sliding mode are very large.

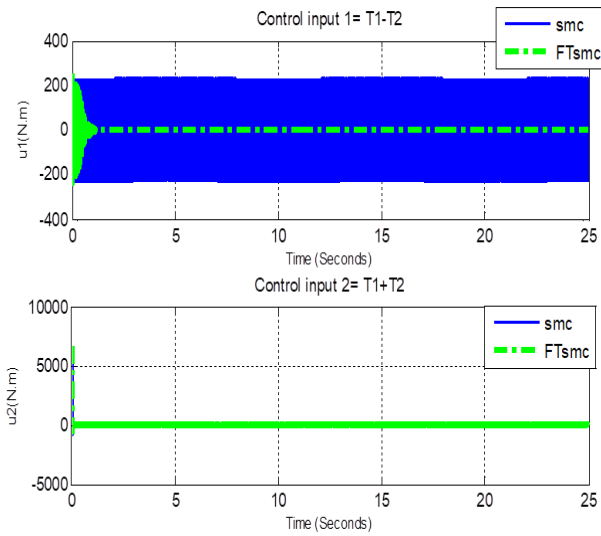


Figure 3.3. Control efforts related to tracking reference paths with the sign function

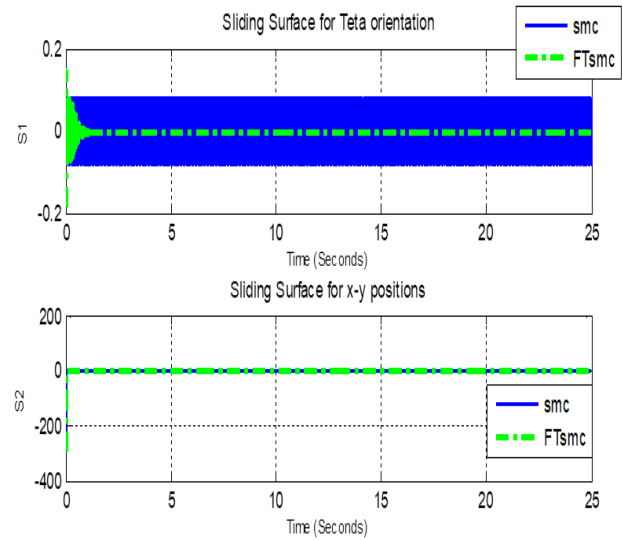


Figure 3.4. Slip surfaces related to tracking reference paths with the sign function

In Figures 3.5 and 3.6, it can be seen that the path tracking in the three-dimensional plane for PD sliding mode control deviates from the path after traveling a certain distance, while for fast terminal sliding mode control, it is accompanied by some persistent error but always remains convergent.

3.1.2 Hyperbolic tangent function control of fast terminal sliding modes

As observed, fluctuations were observed, especially in the control effort diagram, as chattering. One of the commonly proposed strategies to eliminate or reduce these fluctuations is to replace the hyperbolic tangent or saturation function in the control input relation [10, 11]:

$$u_1 = (\ddot{x}_{5d} + (x_6 - \dot{x}_{5d})(c_1 + c_2 \frac{p}{q} (e_{1x})^{\frac{p}{q}-1}))/g_3 + |(-k_1 \tanh + (S_1) - k_2 S_1)/g_3 \quad (28)$$

And

$$u_2 = \frac{-\lambda_1(f_1 - \ddot{x}_{1d} + (x_2 - \dot{x}_{1d})(c_3 + c_4 \frac{p}{q} (e_{2x})^{\frac{p}{q}-1}) - \beta_1(f_2 - \ddot{x}_{3d} + (x_4 - \dot{x}_{3d})(c_5 + c_6 \frac{p}{q} (e_{3x})^{\frac{p}{q}-1}))}{\lambda_1 g_1 + \beta_1 g_2} + (-k_3 \tanh(S_4) - k_4 S_4)/(\lambda_1 g_1 + \beta_1 g_2) \quad (29)$$

Now run the simulation again with the same values as in the previous section and the following graphs are obtained:

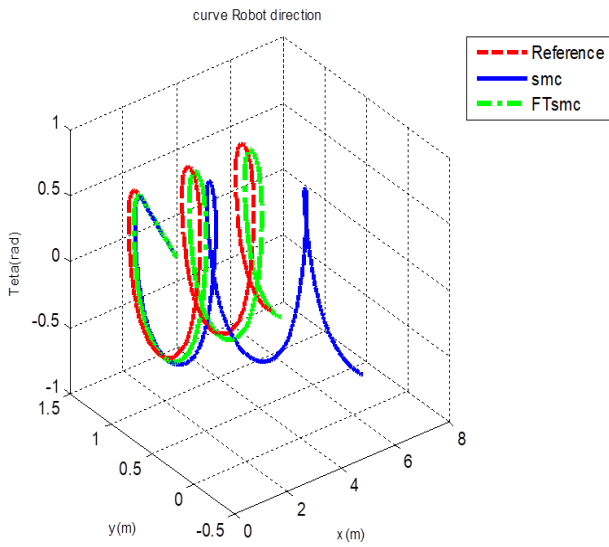


Figure 3.5. Overall tracking path in 3D space with the sign function

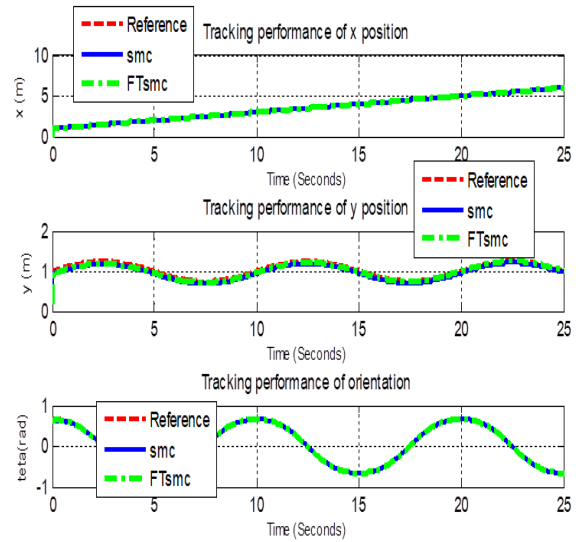


Figure 3.6. Tracking paths related to positions and angles of a wheeled mobile robot with the hyperbolic tangent function

It is observed that the tracking of the positions of x, y and θ angles for the PD sliding mode control and the proposed method are well performed. It is observed that the sinusoidal path tracking in the two-dimensional plane for PD sliding mode control is tracked with persistent error, while for terminal sliding mode control it converges quickly after some time without error. In Figures 3.7 and 3.8, it is observed that in the control effort diagrams, the amplitude of the second input for the proposed method is greater than that of the PD sliding mode, which can be a weakness. Similarly, some momentary changes are seen in the first input. In Figure 3.9, it is observed that in the slip surface diagrams, some momentary changes are seen in the first slip surface in the proposed method.

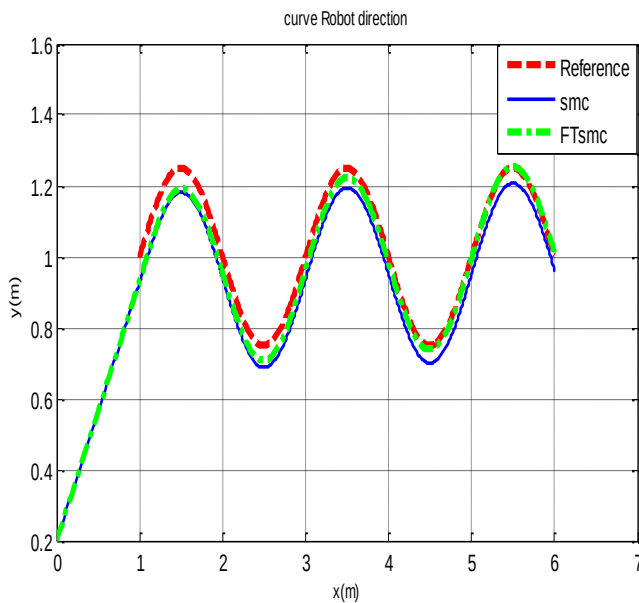


Figure 3.7. Robot tracking in a plane with the hyperbolic tangent function

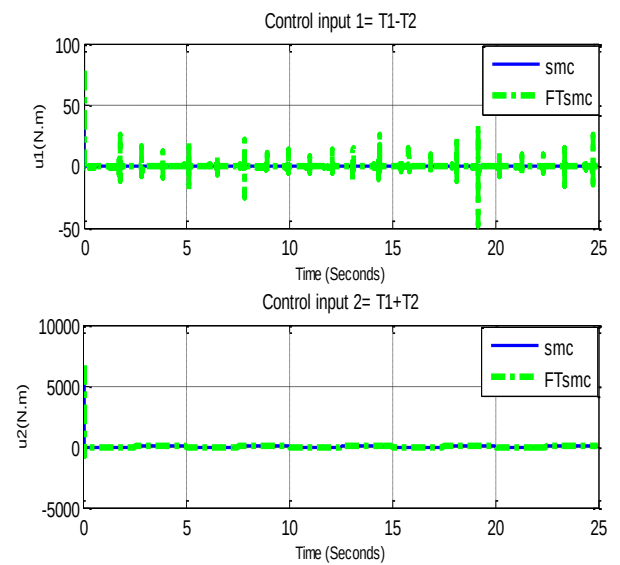


Figure 3.8. Control efforts related to tracking reference paths with the hyperbolic tan-gent

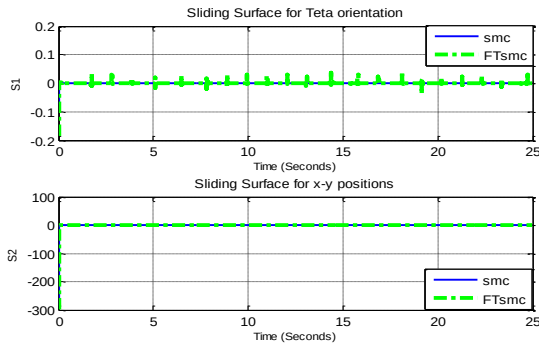


Figure 3.9 - Slip surfaces related to tracking reference paths with the hyperbolic tangent function

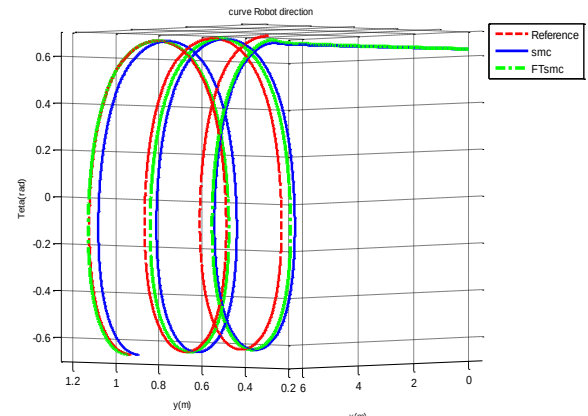


Figure 3.10 - Overall path traced in 3D space with the hyperbolic tangent function

In Figure 3.10, it can be seen that the overall path of movement traveled is always followed with some error in the three-dimensional plane for PD sliding mode control, but it is established error-free for rapid terminal sliding mode control. According to diagram of Figure 3.11, the following point can be explained: *Lyapunov's* stability principle, which states that if a state variable is increasing, its derivative must be decreasing for stability (first and second cases), and if a state variable is decreasing, its derivative must be increasing for stability (third case), which is significant.

3.1.3 Fast terminal sliding mode control in the presence of random noise

Now again, considering the simulation time $T_{sim} = 25\text{ s}$ and sampling time constant $T_s = 0.001$ and the parameter values according to Table 1, $\lambda_1 = 5.5$ and $\beta_1 = 0.5$ and $q = 2$ and $p = 1$. Considering the control input that was created and how they are used in Matlab program, the graphics that follow are taken from:

We introduce an external disturbance in the form of random noise with an amplitude of 0/7 N-m at the input to the system, which can be considered to include an infinite number of frequencies. Then, We look at the suggested controller's resistance to this shock as well as PD's sliding mode control as shown in Figure 3.12 [12, 13].

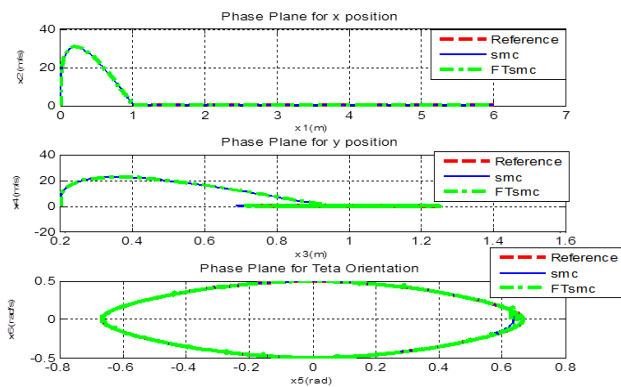


Figure 3.11 Phase plane of system state variables with hyperbolic tangent function

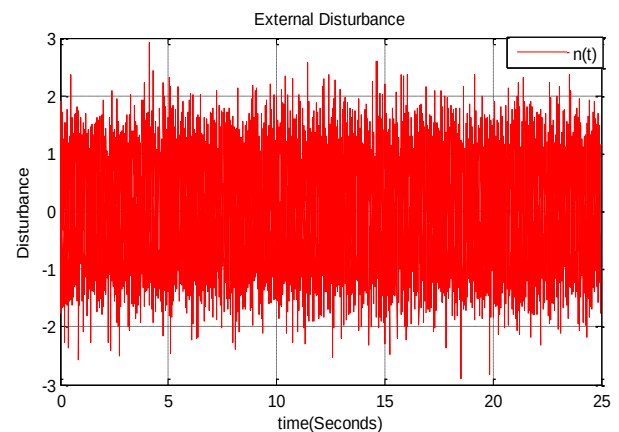


Figure 3.12 Random noise introduced into the robot system

Considering the control input that was with the hyperbolic tangent function and their application in MATLAB software, the graphs listed below are once more extracted:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x) + g_1(x)(u_2 + 0.2\text{randn}(\text{size}(t))) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x) + g_2(x)(u_2 + 0.2\text{randn}(\text{size}(t))) \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = f_3(x) + g_3(x)(u_1 + 0.2\text{randn}(\text{size}(t))) \end{cases} \quad (30)$$

It is observed that the tracking of positions x and angles θ is well done for the suggested approach and the PD sliding mode control, but the tracking of position y is well done for the proposed method while it is deviated in the PD sliding mode control as shown in Figures 3.13 and 3.14.

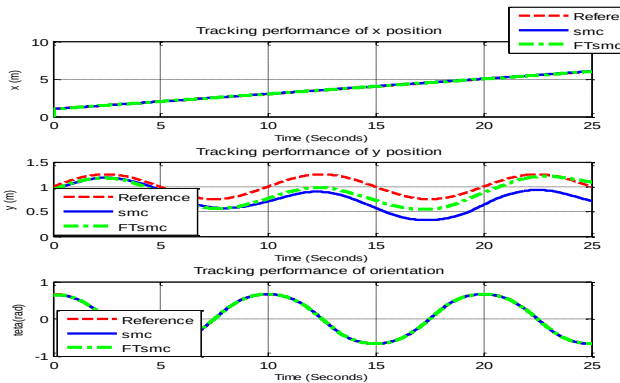


Figure 3.13. Tracking the paths related to the positions and angles of a mobile, wheeled robot with random noise.

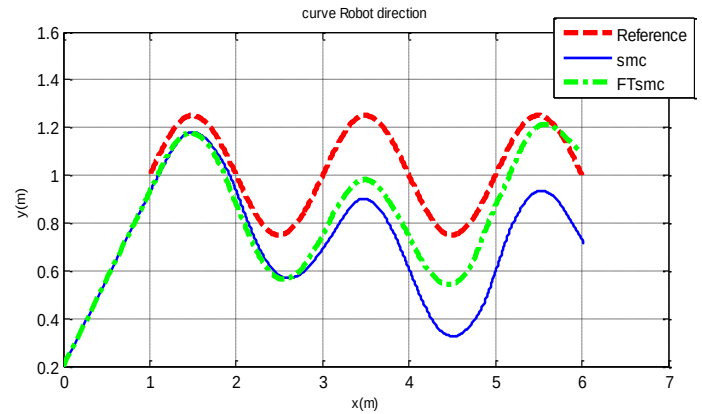


Figure 3.14. Robot Tracking in a Plane with Random Noise

It is observed that the sinusoidal path tracking in a two-dimensional plane for PD sliding mode control is tracked with persistent error while for terminal sliding mode control it converges quickly with less error. According to Figure 3.15, it is observed that the path tracking for PD sliding mode control is tracked with a higher error while for fast terminal sliding mode control it converges with a lower error.

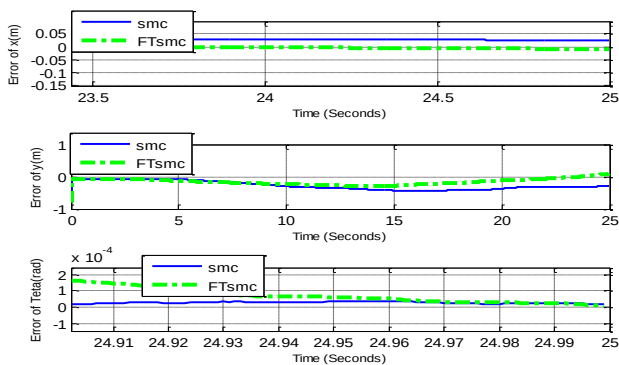


Figure 3.15. Tracking error for reference paths with random noise

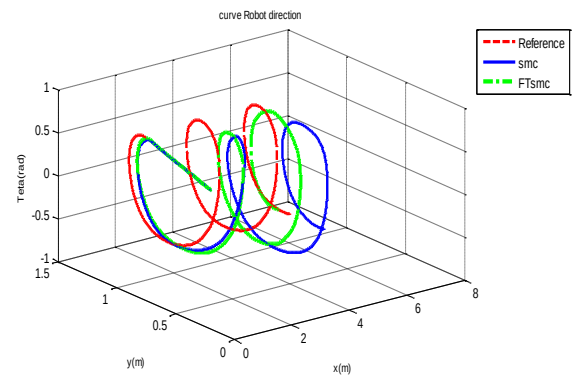


Figure 3.16. Overall tracking path in 3D space with hyperbolic tangent function

In Figure 3.16, the overall motion path can be seen, which shows that tracking the path in the 3D plane for PD sliding mode control is always followed with some error, while for fast terminal sliding mode control it is accompanied by less error. It is observed that the tracking of linear v_x and v_y angular velocities ω_θ for PD sliding mode control is performed with larger fluctuations than the proposed method.

3.1.4 Sliding Mode for the Fast Terminal Defend Against Parametric Variability

Taking the simulation duration into account once more $T_{sim} = 25$ s and sampling time constant $T_s = 0.001$ and the parameter values according to Table 2, $q = 2$ and $p = 1$ and are. In this case, even up to 50% changes are made in parameters such as arm length, arm mass and inertia moment, but the robustness of the proposed controller as well as PD sliding mode control is very good and no noticeable change was observed, of course, the range of required control effort increases significantly [14, 15]:

Table 2. Parameters considered for simulation and control of a wheeled mobile robot with 50% uncertainty applied to the parameters

Robot parameters	Values	Units
m	$10*(1+0/5)$	Kg
J	$3/818*(1+0/5)$	Kg/m ²
r	$0/07*(1+0/5)$	M
L	$0/19*(1+0/5)$	M

To evaluate the performance of the applied control strategy, its performance graph is compared with the sliding mode PD control as shown in Figures 3.18:

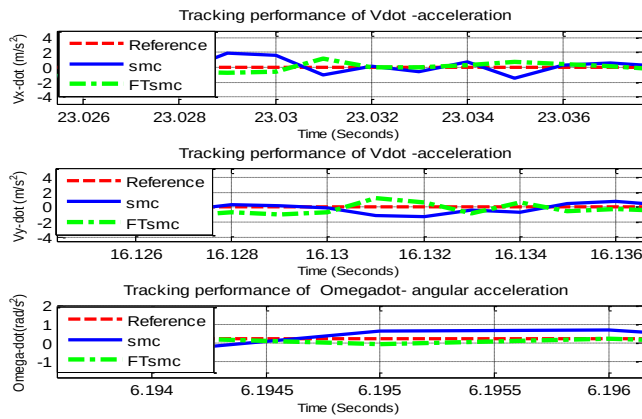


Figure 3.17. Tracking the paths of linear and angular velocities of a wheeled mobile robot with random noise

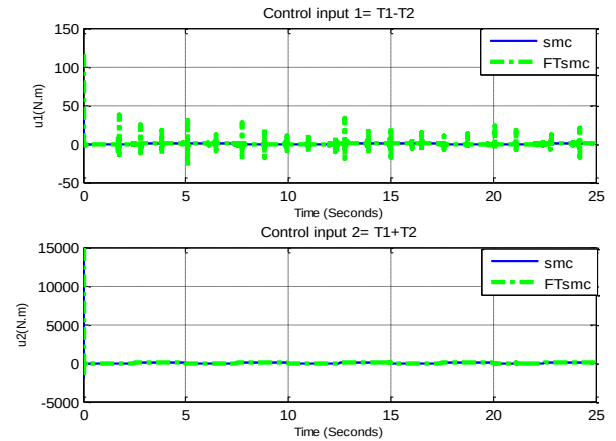


Figure 3.18. Control efforts related to tracking reference paths with 50% parametric uncertainty applied

In Figures 3.17 and 3.18, it is observed that the amount of input amplitude changes at the beginning of time is higher for the proposed method, which is due to the need for faster convergence of the state variables to the desired state of position and velocity. The range of changes of the first control effort is between $(-5, 150)$ Nm and $(-5000, 15000)$ Nm for the second input, which is a large increase and this means that the system tolerates this level of uncertainty, but it will bear a large cost.

3.2 Performance evaluation criteria

To assess the controllers' performance in this investigation, We make use of the following error metrics, which are the average of the sum of the absolute values of the errors [16, 17]:

$$\begin{cases} E_x = \frac{1}{T} \int_0^T (|e_{x_1}| + |e_{x_3}| + |e_{x_5}|) dt \\ E_{v_x} = \frac{1}{T} \int_0^T (|e_{x_2}| + |e_{x_4}| + |e_{x_6}|) dt \end{cases} \quad (31)$$

3.2.1. Performance evaluation in normal mode

Now, we will qualitatively and quantitatively examine the graphs and Table 2 of the system performance in the mode where the saturation function is used, for the tracking error of the robot's position and linear velocity, orientation angle, and angular velocity:

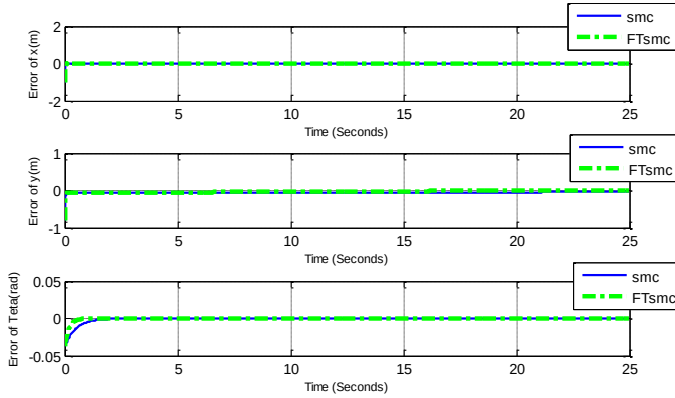


Figure 3.19 Position and status error diagram in normal mode

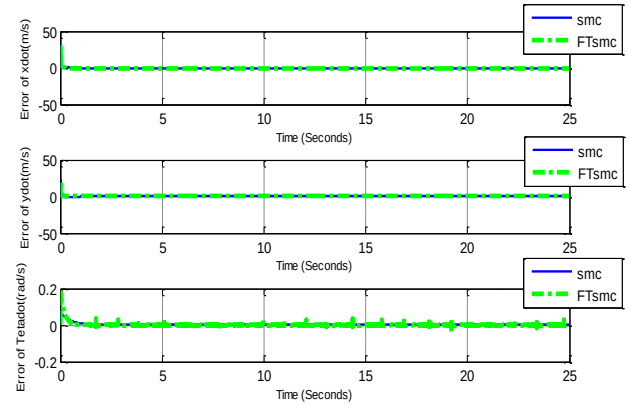


Figure 3.20. Speed error diagram in normal mode.

By observing and examining Figures 3.19 and 3.20, it can be said that the position and velocity tracking error tends to zero and accurate position and velocity tracking is well done. Now, for greater accuracy, the performance quantity of these two controllers is given in Table 3 for the average values of the absolute value integral of the position and velocity error in the time interval of 25 seconds for the two controllers:

Table 3. Average values of the absolute value integral of the position and velocity errors for the fast terminal sliding mode and PD sliding mode controllers

Speed error	Status error	Average integral of absolute value of position and velocity error
0/0754	0/0335	FTSMC
0/0725	0/0635	PDSMC

According to Table 3, it is observed that the average integral value of the absolute value of the position tracking error. When compared to the PD sliding mode control, the error in the rapid terminal sliding mode control is cut in half. However, the average integral value of the absolute value of the speed tracking error in the proposed method is slightly higher. The value of the suggested approach is displayed in this Table.

3.2.2. Performance evaluation in the case of random noise perturbation.

Now, we will qualitatively and quantitatively examine the graphs and Table of the system performance again in the case where the saturation function and random noise disturbance with a height of 0.7 Nm are introduced into the system, for the tracking error of the robot's position and angular velocity [15, 18]:

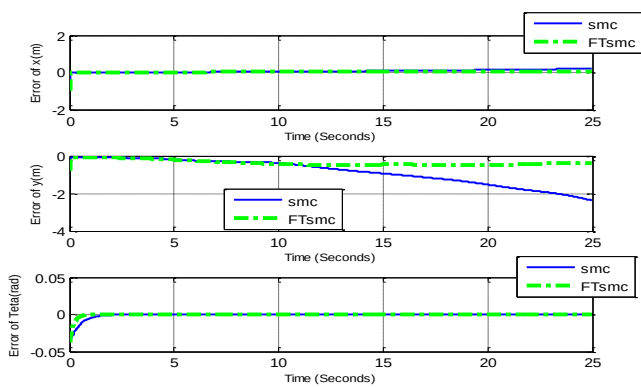


Figure 3.21 - Position and status error diagram in the presence of random noise.

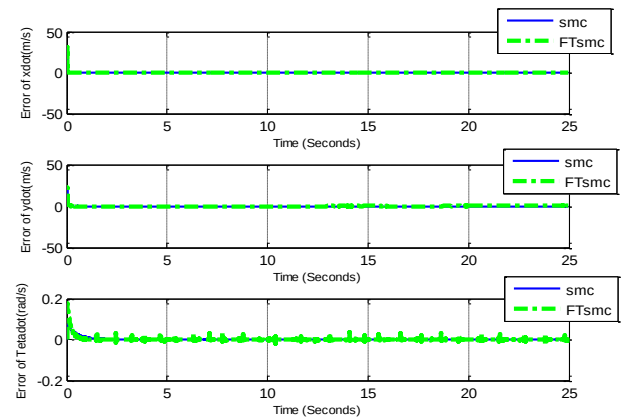


Figure 3.22 Velocity error diagram in the presence of random noise.

By observing and examining the Figures 3.21 and 3.22, it can be said that the effect of this disturbance with the desired amplitude has a negligible effect on the position tracking and tracking speed in the fast terminal sliding mode control, while its effects on the position tracking part on the PD sliding mode control are much greater. It is also observed that the effect of noise on the speed graphs is less. Now, for greater accuracy, the performance quantity of these two controllers is given in Table 4 for the average values of the absolute value integral of the position and speed error in the time interval of 25 seconds for the two controllers:

Table 4. Average values of the absolute magnitude integral of the position and velocity errors for fast terminal sliding mode and PD sliding mode controllers due to random noise disturbance

Speed error	Status error	Average integral of absolute value of position and velocity error
0/1003	0/3955	FTSMC
0/1722	0/9127	PDSMC

According to Table 4, it is observed that the average integral value of the absolute value of the tracking error in the fast terminal sliding mode control is reduced to about one third compared to the PD sliding mode control. In addition, unlike the noiseless case of Table 4, the average integral value of the absolute value of the tracking error in the proposed method is less than that of the PD sliding mode control. Of course, in both methods, the magnitude of the tracking error increases significantly compared to the conventional case, which limits the controllers.

3.3 Discussion

Finally, by investigating some graphs, it can be seen that chattering effects are more effective in the derivative of states (for example, control inputs). which according to the relation of derivation in the Fourier transform, which indicates that the act of time derivation means the linear amplification of system frequencies. It can be interpreted as meaning that higher frequencies get more amplification.

The non-holonomic wheeled mobile robot system is a system in which the number of inputs is less than the number of controlled state variables, which are also called under-excitation systems. From the control point of view, when the dynamic equations of non-holonomic systems are written in the state space form, they have fewer inputs than the number of state variables.

For this reason, the issue of stabilizing them, or in other words, designing a regulator for them, is a problem for which a precise solution has not been proposed so far. Although it is proven that non-holonomic systems can be controllable despite having fewer inputs than the number of state variables, it is proved that finding continuous inputs that are only a function of the error of the state variables for stabilization These systems are impossible.

This problem makes the control system design for non-holonomic systems very difficult. As seen, the fast terminal sliding mode control, which is a non-linear controller, is well suited to the control of this non-linear robot.

In addition, although the control system has good robustness against the external disturbances and parametric uncertainties, at the same time, the error values and the range of the control inputs indicate the limitation of the amount of imposed external disturbances or the percentage of allowed parametric uncertainty.

4. Conclusions

The non-holonomic wheeled mobile robot system is a system in which the number of inputs is less than the number of controlled state variables, which are also called under-excitation systems. From the control point of view, when the dynamic equations of non-holonomic systems are written in state space form, they have fewer inputs than the number of state variables. For this reason, the issue of stabilizing them, or in other words, designing a regulator for them, is a problem for which no exact solution has been proposed so far. Although it is proved that non-holonomic systems can be controllable despite having fewer inputs than the number of state variables, it is proved that finding continuous inputs that are only a function of the error of the state variables for stabilization These systems are impossible. This problem makes the design of the control system for non-holonomic systems very difficult. But as seen, the fast terminal sliding mode control, which is itself a non-linear controller, is a good response to the control of this non-linear robot. In addition, although the control system has good resistance against external disturbances and parametric uncertainties, at the same time, the error values and the range of the control input indicate the limit of the amount of imposed external disturbances or the percentage of allowed parametric uncertainty. In the end, it is reminded again that the sliding mode is among the most often used controllers. for managing non-linear processes and systems. This acceptance is due to its relative simplicity and resistance to external disruptions and system uncertainty. The issue that the field is facing of its application is the occurrence of the phenomenon of sliding modes on the surface that slides, which is especially noticeable in systems that have fast dynamics. As seen, to solve or reduce this problem, replacing the hyperbolic tangent function instead of the sign function in the switching control part was used as a suitable option, so that this combination softened the behavior and reduced system vibrations.

Conflicts of Interest: "The authors declare no conflict of interest."

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